

Politecnico di Bari



Dipartimento di Ingegneria Elettrica e dell'Informazione Corso di Laurea Magistrale in Ingegneria dei Sistemi Medicali

Bioinformatica Avanzata

RNA-Seq

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Outline

- Next-generation sequencing
- Sequencing strategies
- TCGA repository
- Differential expression analysis
- Negative binomial model
- Example: IncRNAs in prostate cancer in relation to Gleason score

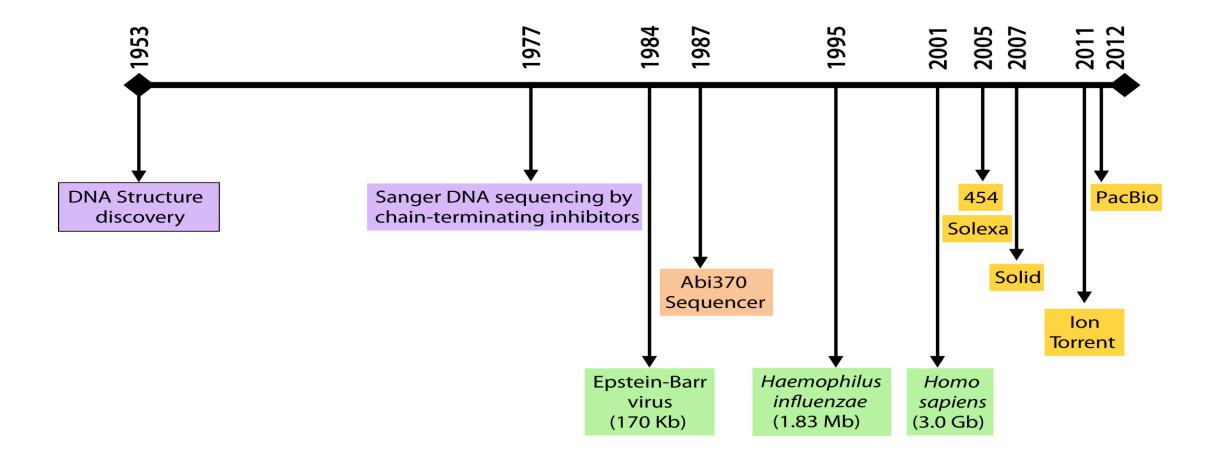








DNA sequencing timeline



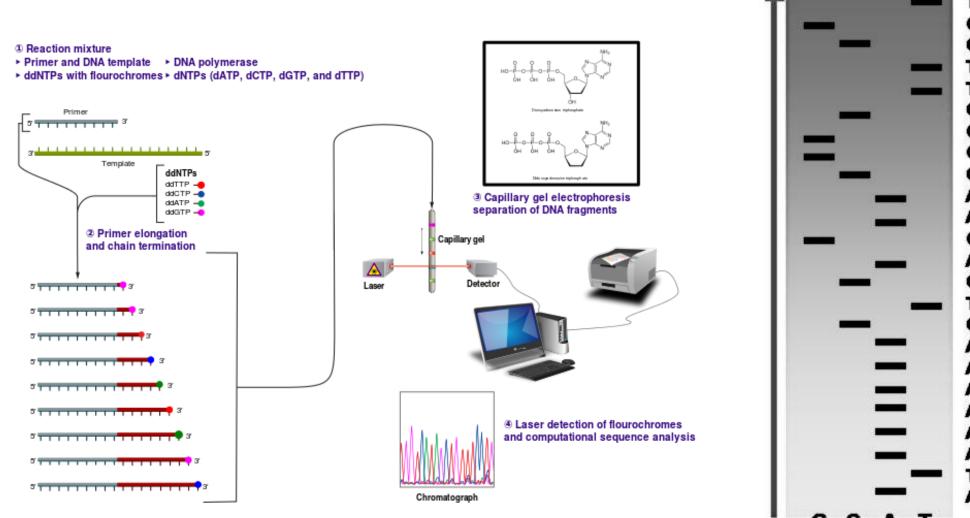


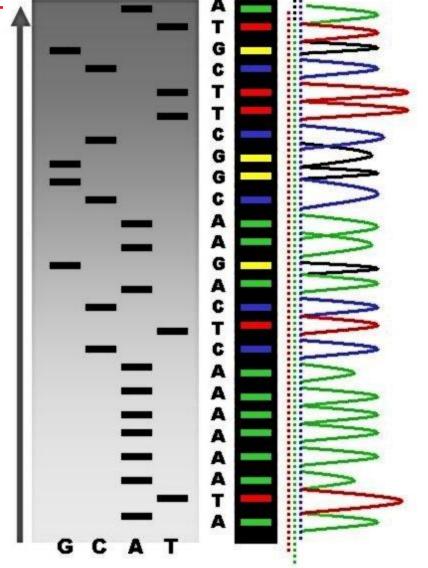






Sanger method













Sanger sequencing reached its technical limits

- Only modestly parallel (394 lanes/machine)
- Long read lengths (500-900 bp) & >99.9% correct
- Need to clone the DNA to obtain enough for sequencing reaction

 cost for typical Sanger sequencing is \$5-6/sample with reliable 500 bp of sequence

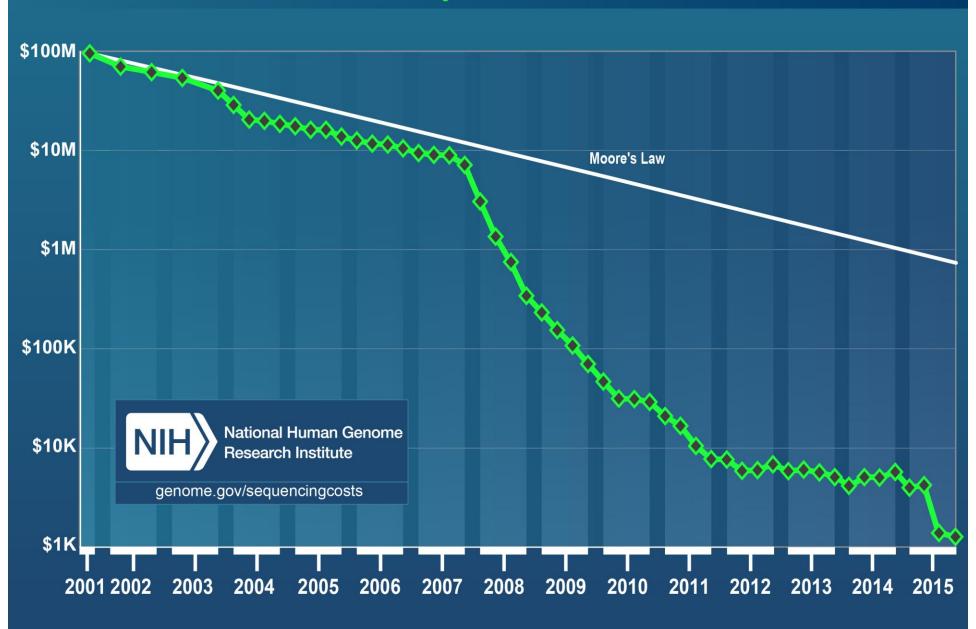


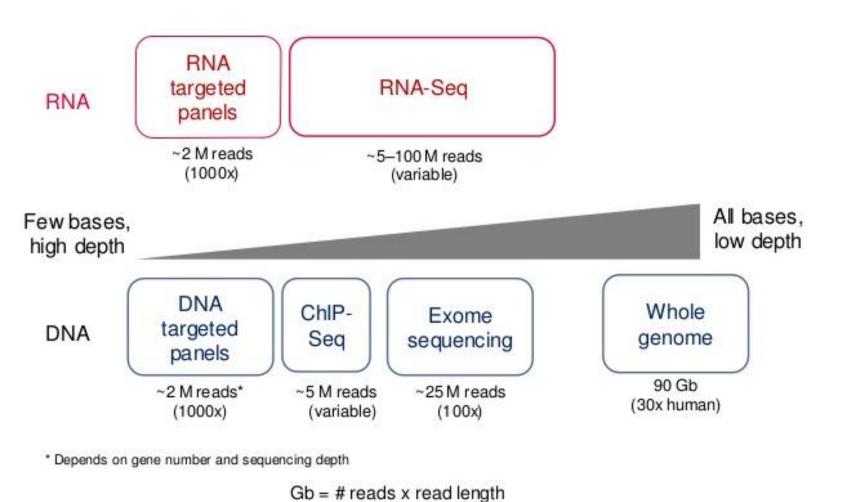


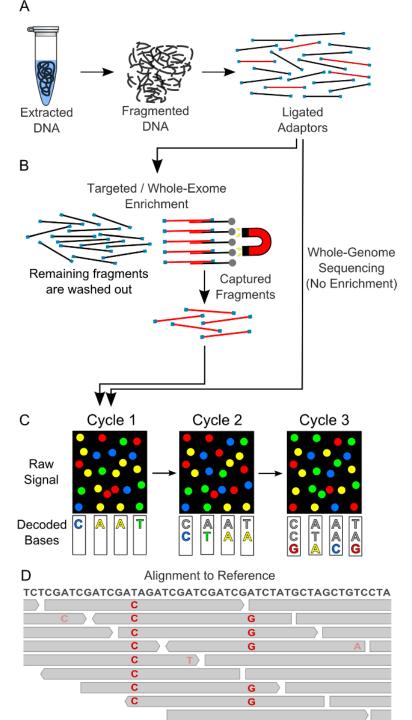




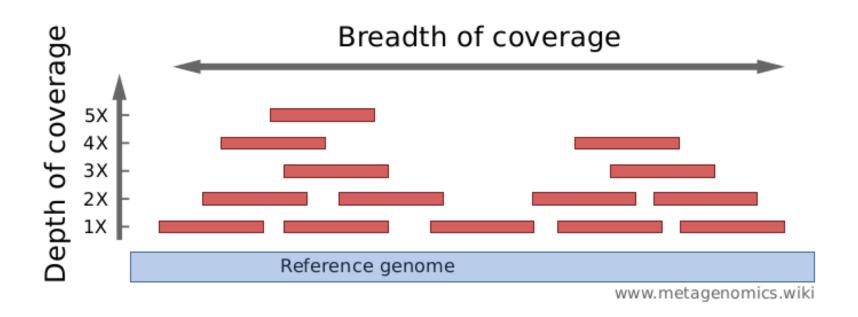
Cost per Genome







Important sequencing parameters

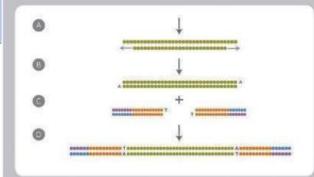


Library Prep:

~ 6 hours



Illumina NGS

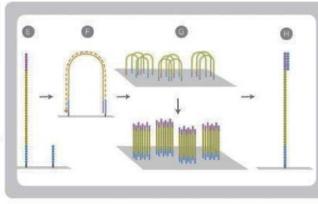


- A) Fragment DNA
- B) Repair ends/Add A overhang DNA
- C) Ligate adapters
- D) Select ligated DNA

Cluster generation

~ 6 hours



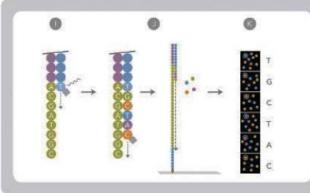


- E) Attach DNA to flow cell
- F) Bridge amplification
- G) Generate clusters
- H) Anneal sequencing primer

Sequencing

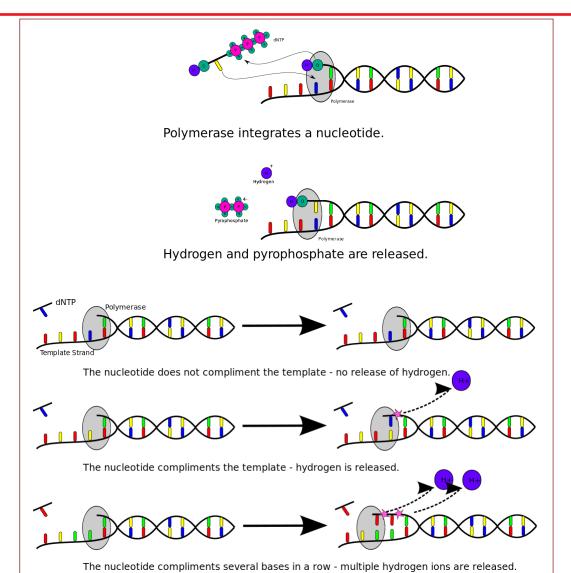
2-6 days





- I) Extend 1st base, read & deblock
- K) Generate base calls
- J) Repeat to extend strand

Ion Torrent – measures pH changes



Done on a semiconductor chip

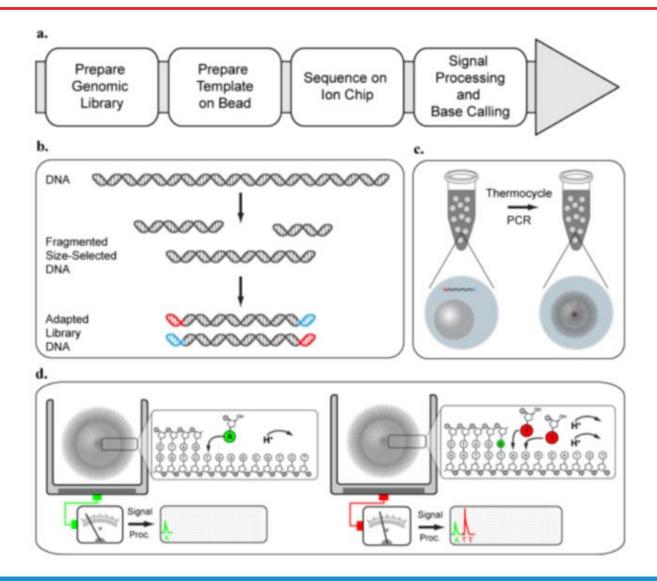








Ion Torrent workflow



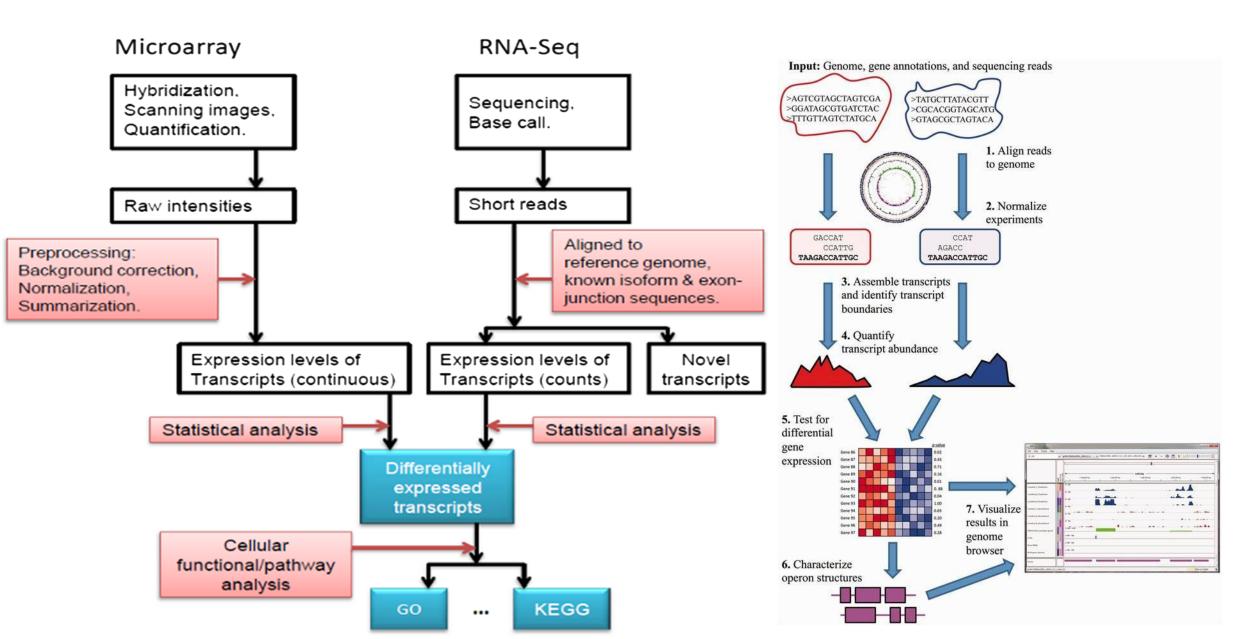




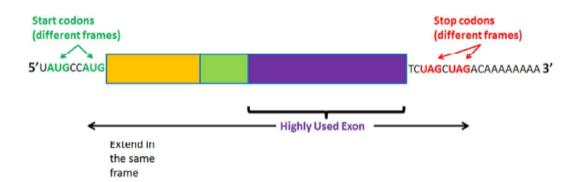


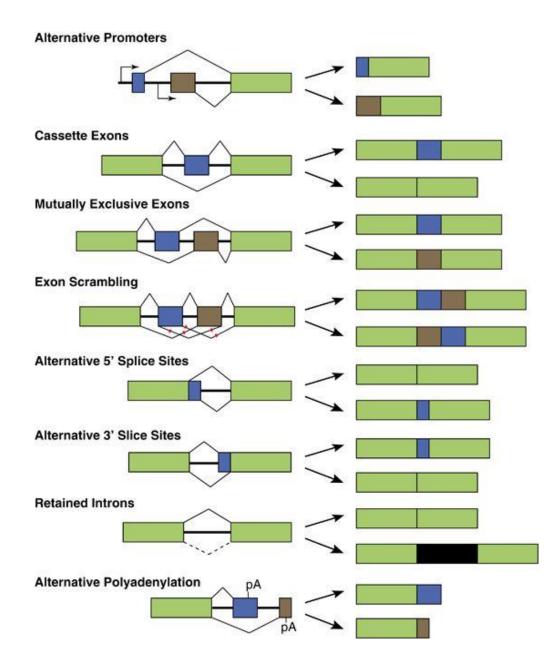


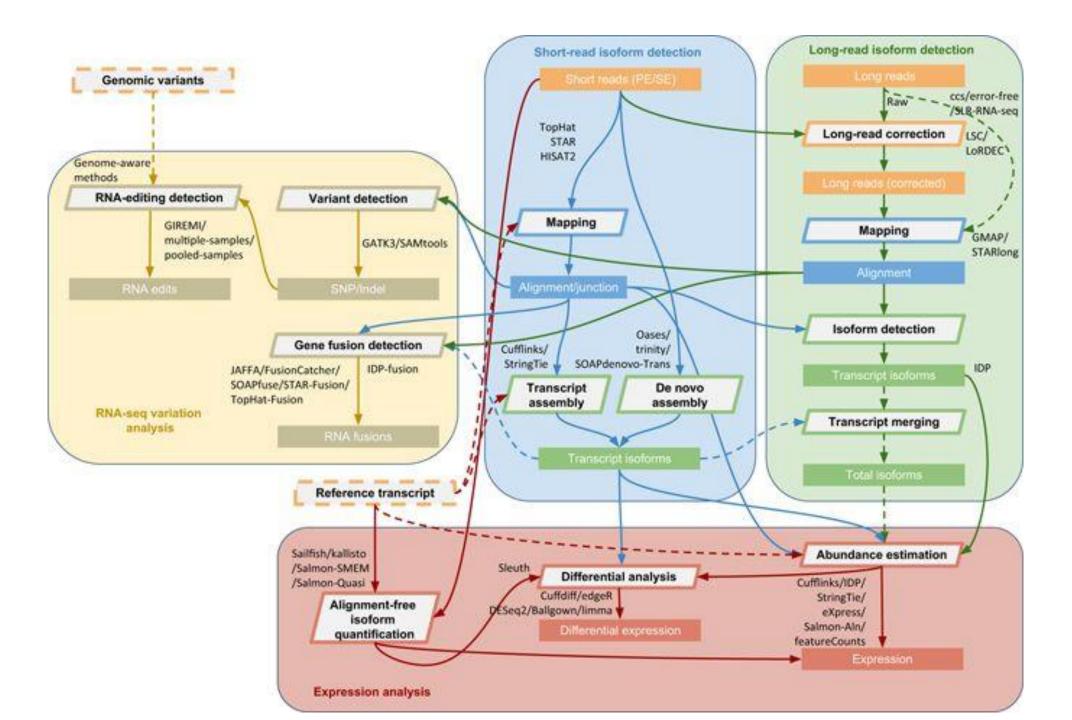
RNA-Seq: a powerful approach



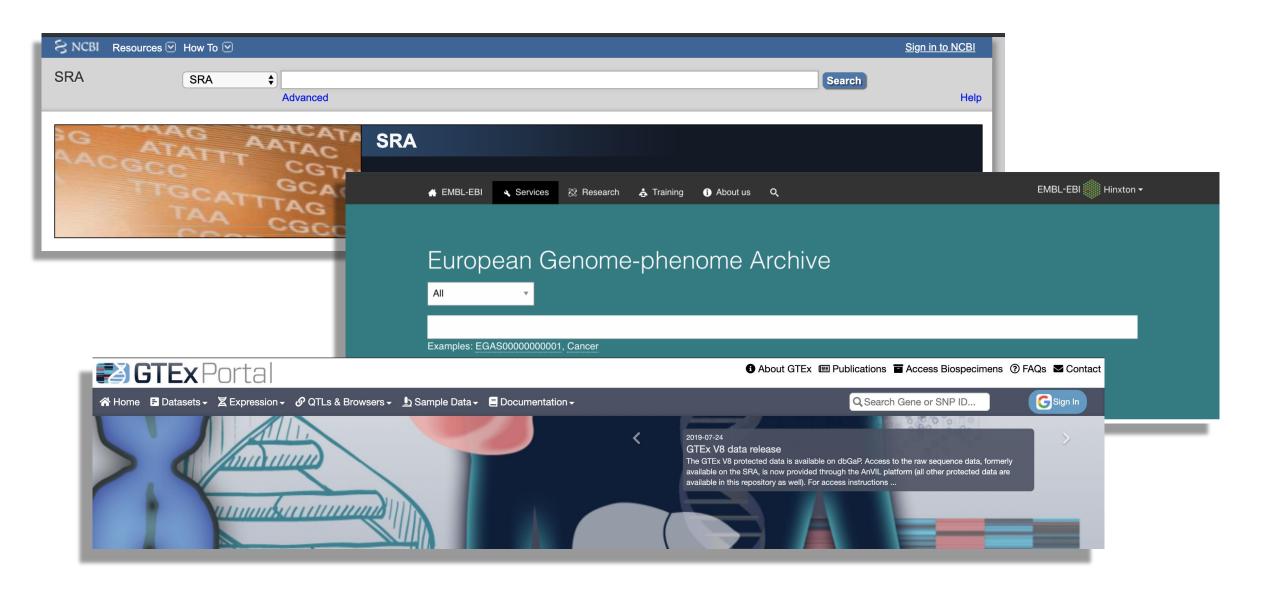
Transcript isoforms







Repositories

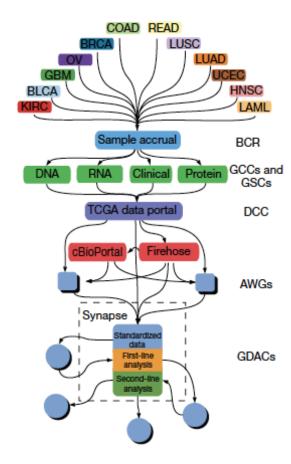


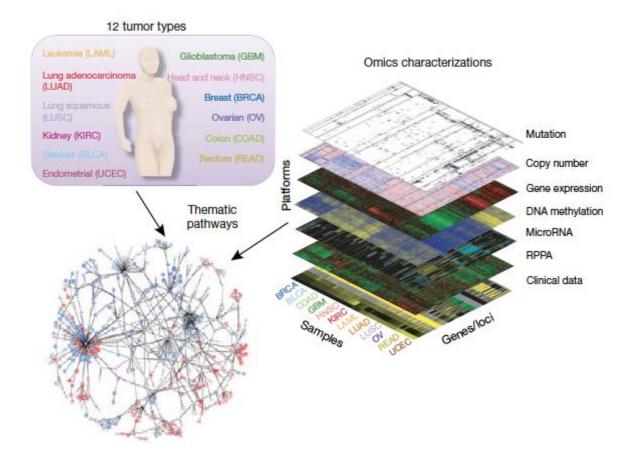


The Cancer Genome Atlas Pan-Cancer analysis project

The Cancer Genome Atlas Research Network¹, John N Weinstein^{2,3}, Eric A Collisson⁴, Gordon B Mills³, Kenna R Mills Shaw^{5,6}, Brad A Ozenberger⁷, Kyle Ellrott^{8,9}, Ilya Shmulevich¹⁰, Chris Sander¹¹ & Joshua M Stuart^{8,9}

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NATIONAL CANCER INSTITUTE THE CANCER GENOME ATLAS

TCGA BY THE NUMBERS

TCGA produced over of data

To put this into perspective, 1 petabyte of data is equal to

212,000

TCGA data describes

DIFFERENT TUMOR TYPES ...including

RARE CANCERS

...based on paired tumor and normal tissue sets collected from



DIFFERENT DATA TYPES



TCGA RESULTS & FINDINGS



MOLECULAR BASIS OF CANCER

Improved our understanding of the genomic underpinnings of cancer

For example, a TCGA study found the basal-like subtype of breast cancer to be similar to the serous subtype of ovarian cancer on a molecular level, suggesting that despite arising from different tissues in the body, these subtypes may share a common path of development and respond to similar therapeutic strategies.



TUMOR SUBTYPES

Revolutionized how cancer is classified

TCGA revolutionized how cancer is classified by identifying tumor subtypes with distinct sets of genomic alterations.*



THERAPEUTIC **TARGETS**

Identified genomic characteristics of tumors that can be targeted with currently available therapies or used to help with drug development

TCGA's identification of targetable genomic alterations in lung squamous cell carcinoma led to NCI's Lung-MAP Trial, which will treat patients based on the specific genomic changes in their tumor.

THE TEAM



and Canada

WHAT'S NEXT?

The Genomic Data Commons (GDC) houses TCGA and other NCI-generated data sets for scientists to access from anywhere. The GDC also has many expanded capabilities that will allow researchers to answer more clinically relevant questions with increased ease.

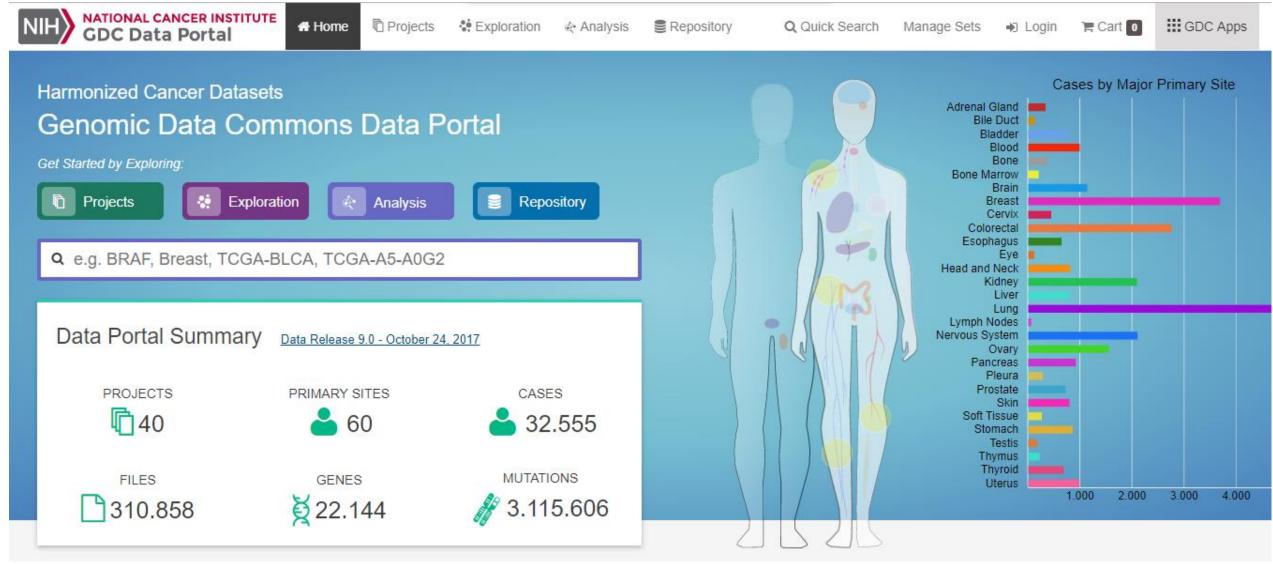


*TCGA's analysis of stomach cancer revealed that it is not a single disease, but a disease composed of four subtypes, including a new subtype characterized by infection with Epstein-Barr virus.

What we have....

The Cancer Genome Atlas (TCGA)

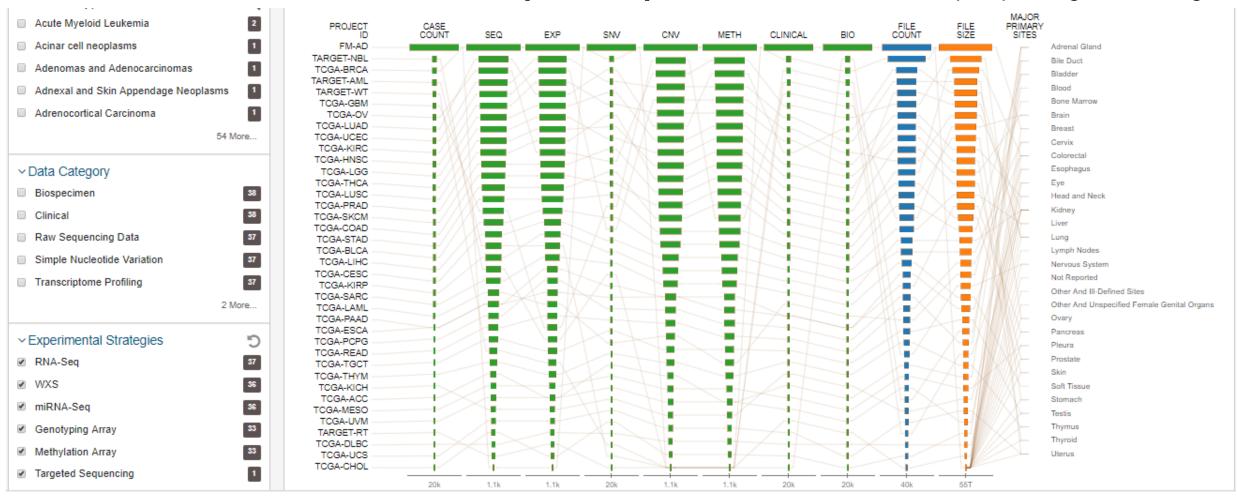
https://portal.gdc.cancer.gov/



What we have....

The Cancer Genome Atlas (TCGA)

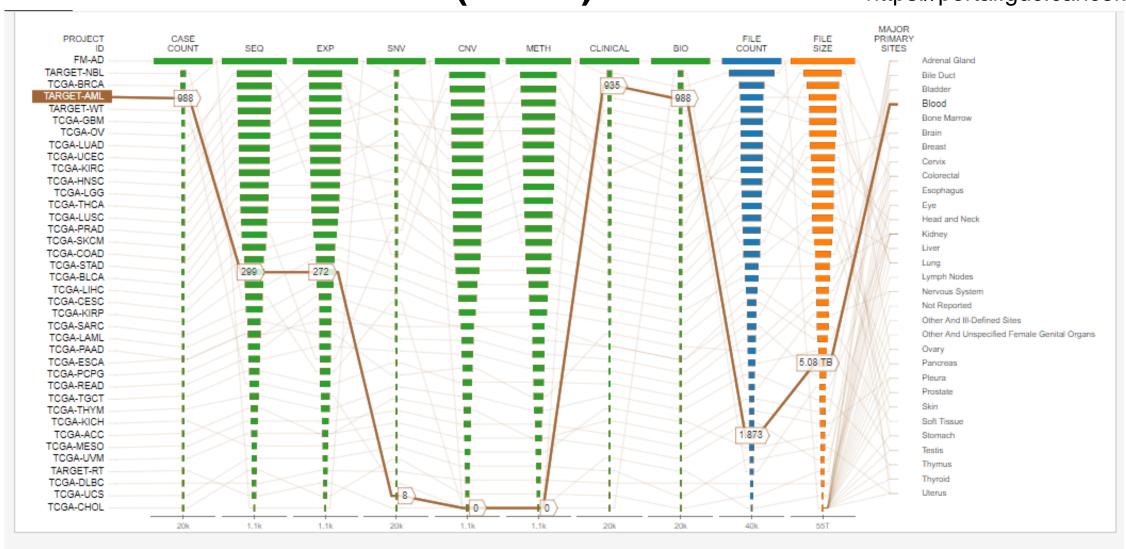
https://portal.gdc.cancer.gov/

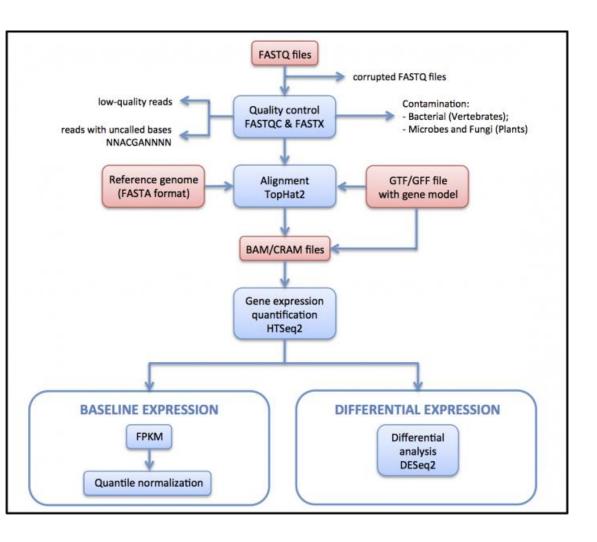


What we have....

The Cancer Genome Atlas (TCGA)

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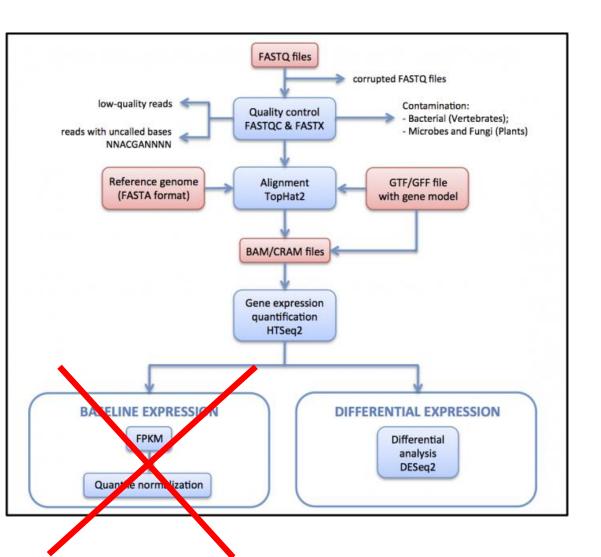


Examples

 $RPKM = \#MappedReads * \frac{1000bases * 10^6}{length \ of \ transcript * Total \ number \ of \ mapped \ reads}$

- Example 1:
 - 2500kb transcript with 900 alignments in a sample of 10 million reads (out of which 8 million reads can be mapped):

• RPKM=900 *
$$\frac{1000 * 10^6}{2500 * 8.10^6}$$
 = 45



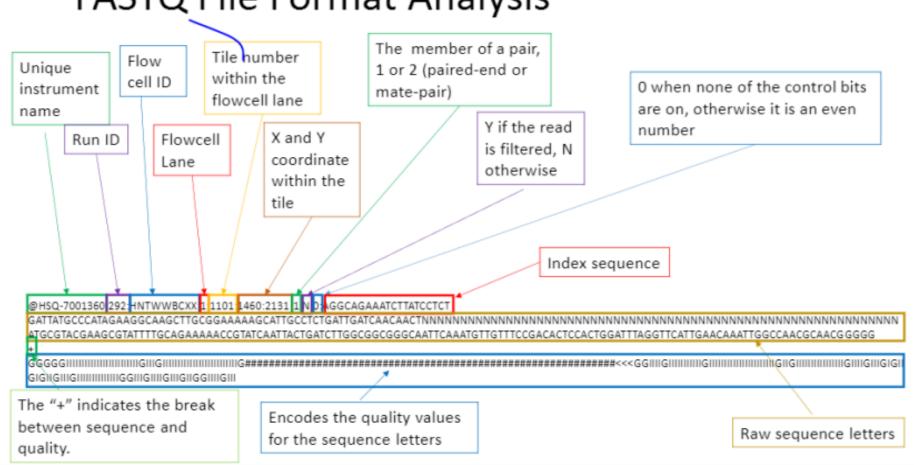
An update (12th August 2018):

You should abandon RPKM / FPKM normalisation. They are not ideal where cross-sample differential expression analysis is your aim; indeed, they render samples incomparable via differential expression analysis: Please read this: A comprehensive evaluation of normalization methods for Illumina high-throughput RNA sequencing data analysis

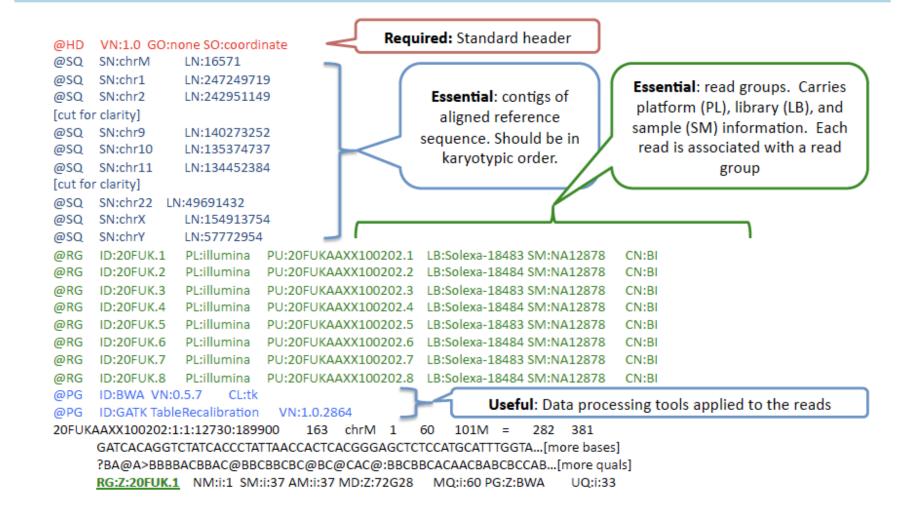
In their key points:

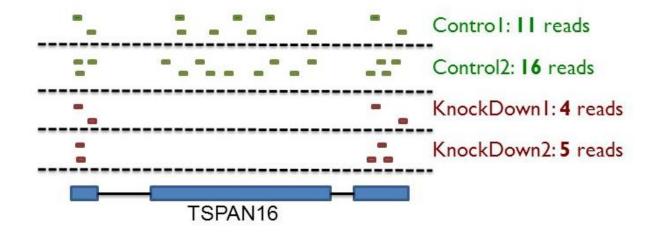
The Total Count and RPKM normalization methods, both of which are still widely in use, are ineffective and should be definitively abandoned in the context of differential analysis.

FASTQ File Format Analysis



BAM headers: an essential part of a BAM file





 Reads (BAM file) are counted for each gene model (gtf file) using HTSeq-count:

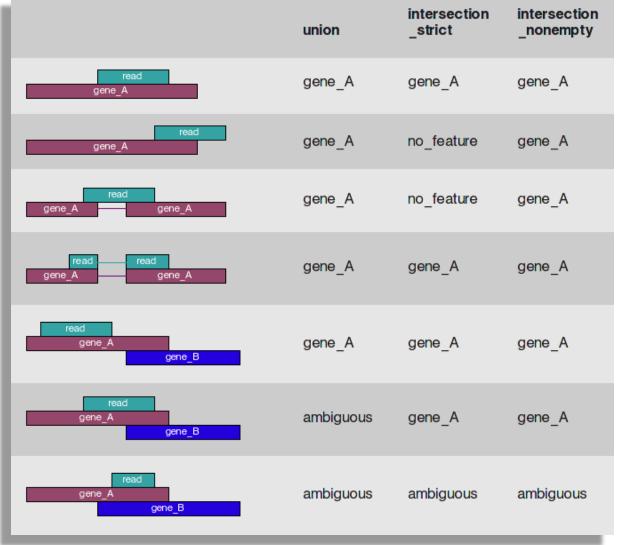
	Control1	Control2	KnockDown1	KnockDown2
TSPAN6	11	16	4	5
TNMD	1	0	0	0
DPM1	435	743	836	739
SCYL3	203	218	416	352
Clorf112	216	643	714	704
FGR	2365	5011	2828	2294
CFH	6	1	4	0
FUCA2	380	865	431	523

NFYA	888	827	1674	1580



www-huber.embl.de/users/anders/HTSeq





For each position i in the read, a set S(i) is defined as the set of all features overlapping position i. Then, consider the set S, which is (with i running through all position within the read)

- the union of all the sets *S(i)* for mode union.
- the intersection of all the sets S(i) for mode intersection-strict.
- the intersection of all non-empty sets S(i) for mode intersection-nonempty.

Sequencing count data

Sequencing counting rules

	control-1	control-2	control-3	treated-1
FBgn0000008	78	46	43	47
FBgn0000014	2	0	0	0
FBgn0000015	1	0	1	0
FBgn0000017	3187	1672	1859	2445
FBgn0000018	369	150	176	288
11				

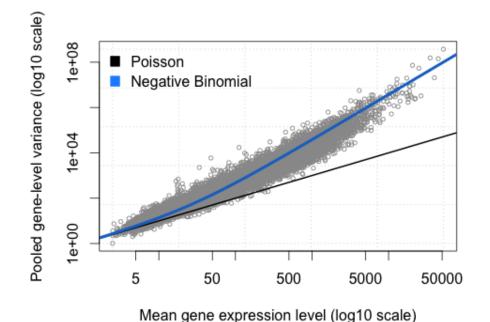
- Count reads, not base-pairs
- Count each read at most once.
- Discard a read if
 - it cannot be uniquely mapped
 - its alignment overlaps with several genes
 - the alignment quality score is bad
 - (for paired-end reads) the mates do not map to the same gene

In RNA-Seq, noise (and hence power) depends on count level.

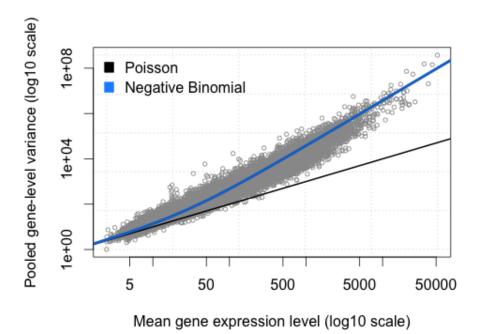
Why?

A FIRST INTUITION

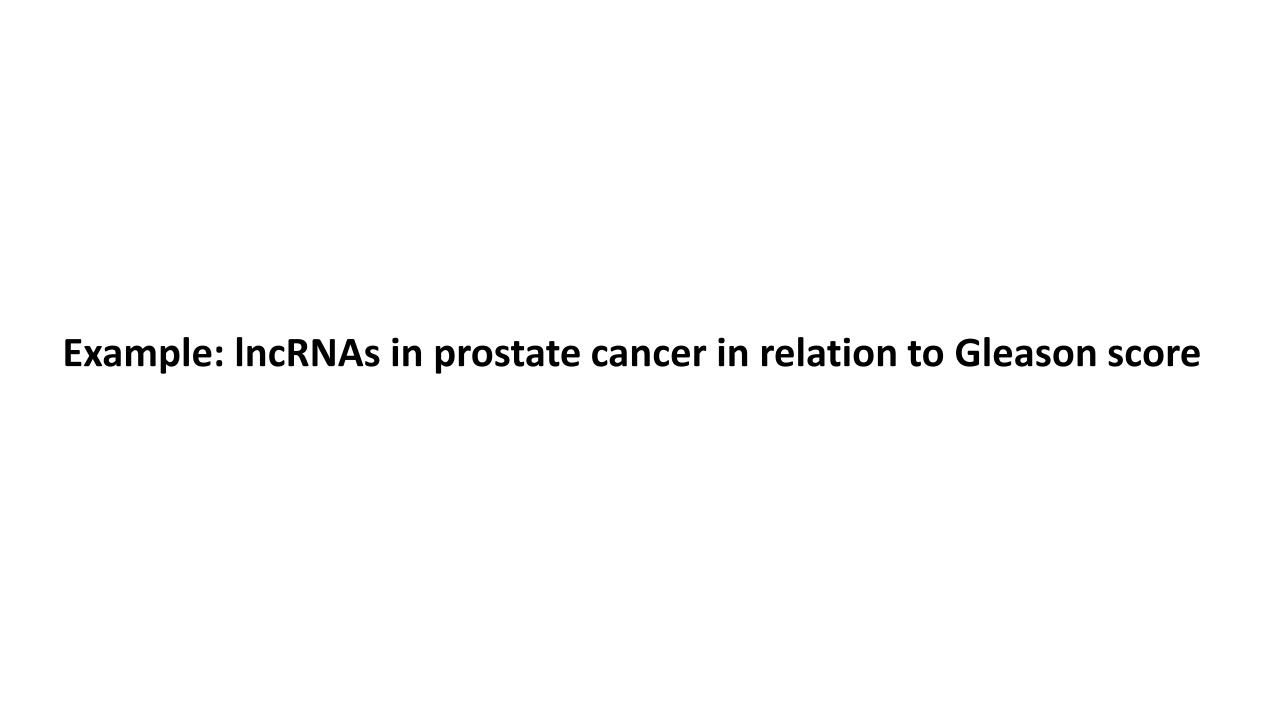
- 1. In a standard sequencing experiment (RNA-Seq), we map the sequencing reads to the reference genome and count how many reads fall within a given gene (or exon).
- 2. This means that the input for the statistical analysis are discrete non-negative integers ("counts") for each gene in each sample.
- 3. The total number of reads for each sample tends to be in the millions, while the counts per gene vary considerably but tend to be in the tens, hundreds or thousands. Therefore, the chance of a given read to be mapped to any specific gene is rather small.
- 4. Discrete events that are sampled out of a large pool with low probability sounds very much like a **Poisson process**. And indeed it is. In fact, earlier iterations of RNA-Seq analysis modeled sequencing data as a Poisson distribution. There is one problem, however. The **variability of read counts** in sequencing experiments tends to be **larger than the Poisson distribution allows**.



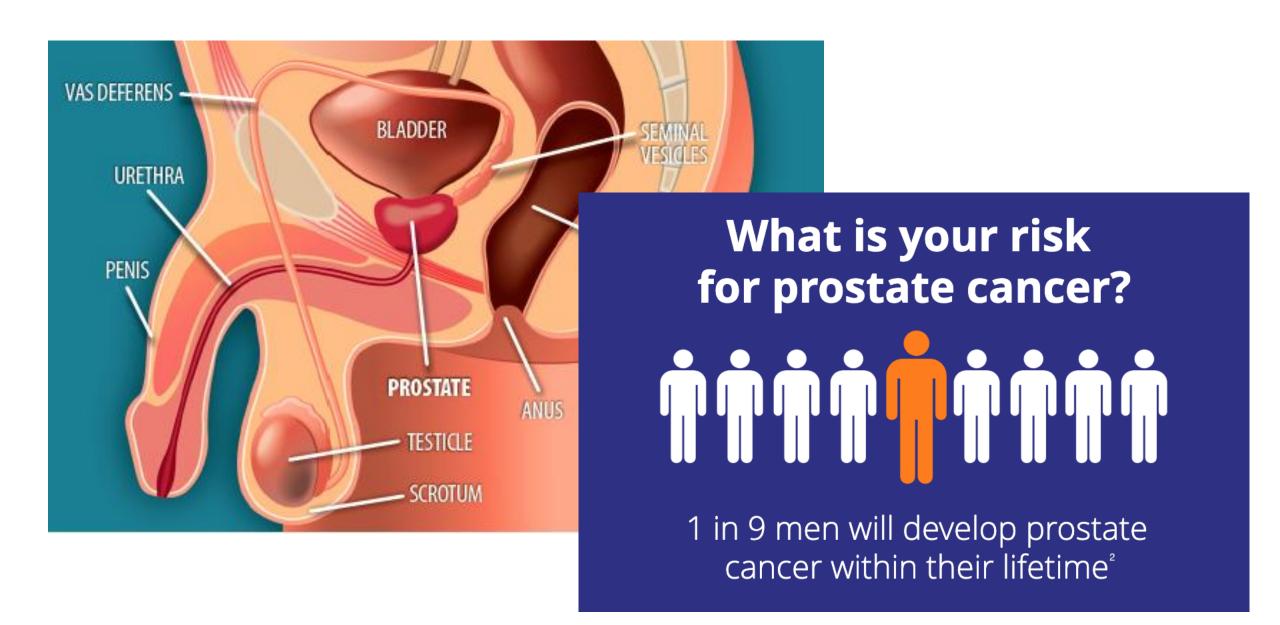
If we assume that our samples are biological replicates, it is not surprising that the **same transcript is present at slightly different levels in each sample**, even under the same conditions. In other words, the Poisson process in each sample has a slightly different expected count parameter. This is the source of the "extra" variance (overdispersion) we observe in sequencing data. In the framework of the NB distribution, it is accounted for by allowing Gamma-distributed uncertainty about the expected counts (the Poisson rate) for each gene.



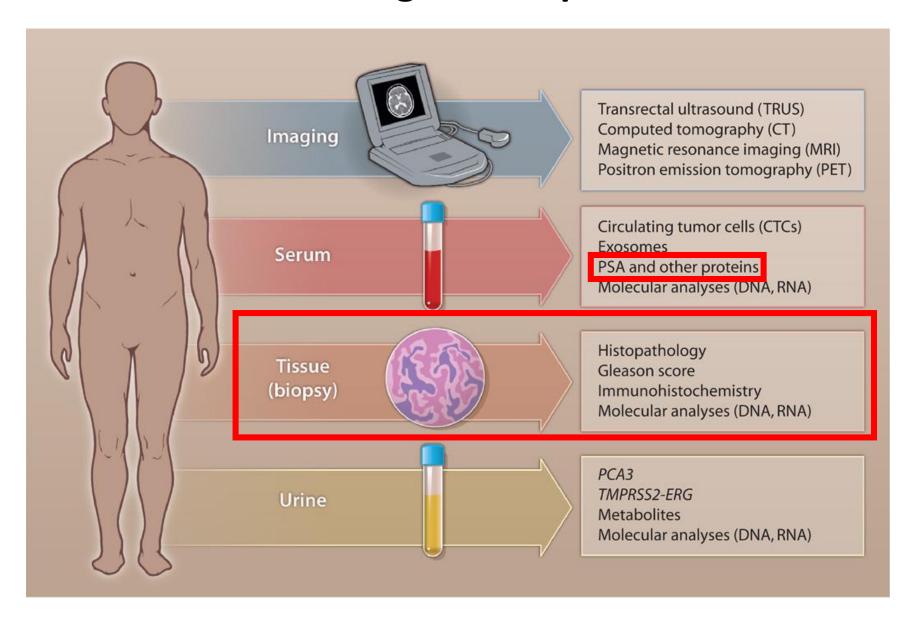
It is obvious that the variance of counts is generally greater than their mean, especially for genes expressed at a higher level. This phenomenon is called "overdispersion". The NB distribution is similar to a Poisson distribution but has an extra parameter called the "clumping" or "dispersion" parameter. It is like a Poisson distribution with more variance.

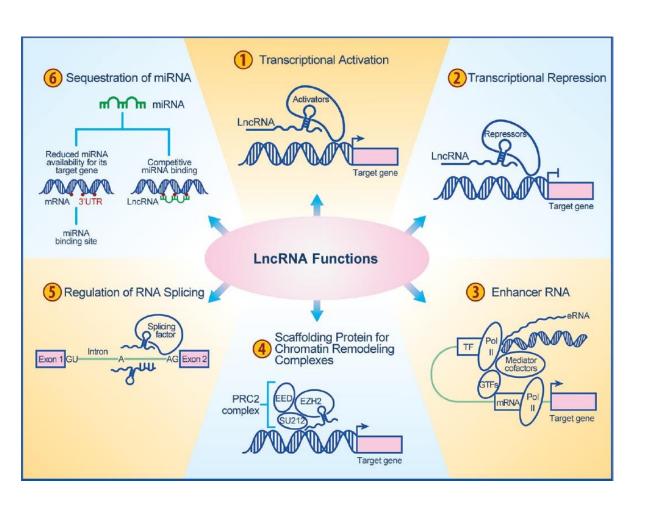


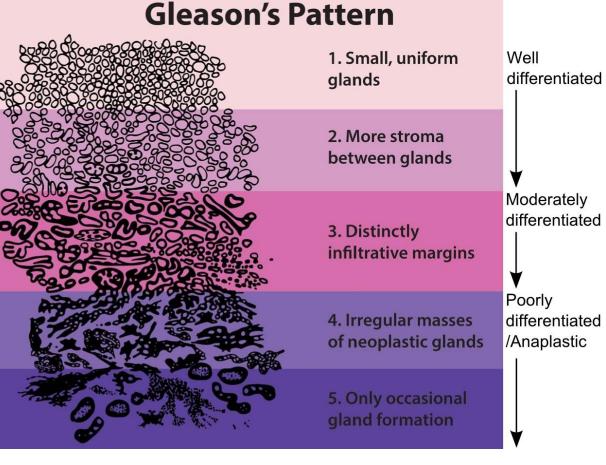
Prostate cancer

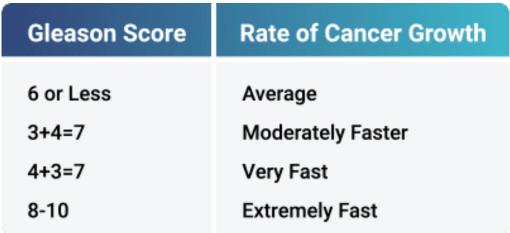


Prostate cancer diagnostics: present and future

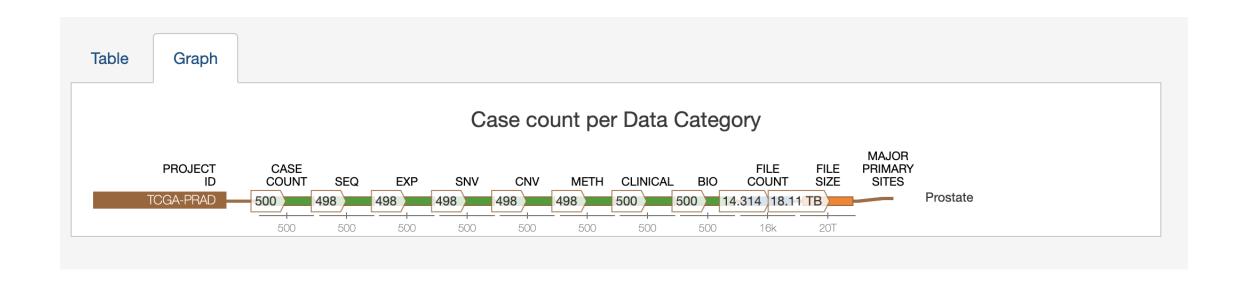








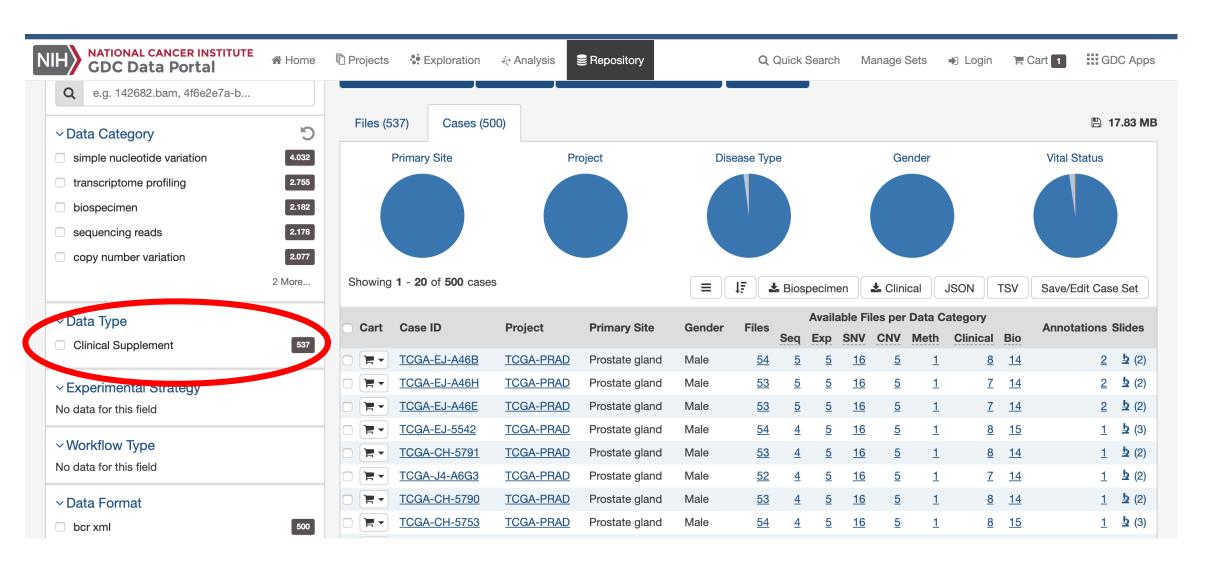
Project: TCGA-PRAD (Prostate Adenocarcinoma)



Project: TCGA-PRAD (Prostate Adenocarcinoma)

H NATIONAL CANCER INSTITUTE GDC Data Portal	☆ Home	© P	rojects	∜ Exploration	﴿∙ Analysis	Repository		Q	Quick	Search	M	anage \$	Sets	→) Login	= (Cart 1 GDC App				
	2 More	s	howing	1 - 20 of 498 case	S			Į₹ .	Bios	pecime	en	≛ Clini	cal	JSON	TSV	Save/Edit Case Set				
∨ Data Type			Cart	Case ID	Project	Primary Site	Gender	Files	Seq			es per		ategory Clinical	Bio	Annotations Slides				
Gene Expression Quantification	1.653		-	TCGA-EJ-A46B	TCGA-PRAD	Prostate gland	Male	<u>54</u>	<u>5</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>8</u>	<u>14</u>	<u>2</u> <u>4</u> (2)				
☐ Isoform Expression Quantification	551			F •	TCGA-EJ-A46H	TCGA-PRAD	Prostate gland	Male	<u>53</u>	<u>5</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>7</u>	<u>14</u>	<u>2</u> ½ (2)			
miRNA Expression Quantification			F	TCGA-EJ-A46E	TCGA-PRAD	Prostate gland	Male	<u>53</u>	<u>5</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>7</u>	<u>14</u>	<u>2</u> ½ (2)				
∨ Experimental Strategy			-	TCGA-EJ-5542	TCGA-PRAD	Prostate gland	Male	<u>54</u>	<u>4</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>8</u>	<u>15</u>	<u>1</u> ½ (3)				
□ RNA-Seq	1.653			TCGA-CH-5791	TCGA-PRAD	Prostate gland	Male	<u>53</u>	<u>4</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>8</u>	<u>14</u>	<u>1</u> 🖢 (2)				
□ miRNA-Seq	1.102		-	TCGA-J4-A6G3	TCGA-PRAD	Prostate gland	Male	<u>52</u>	<u>4</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	7	<u>14</u>	<u>1</u> ½ (2)				
			=	TCGA-CH-5790	TCGA-PRAD	Prostate gland	Male	<u>53</u>	<u>4</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>8</u>	<u>14</u>	<u>1</u> ½ (2)				
∨ Workflow Type			F •	TCGA-CH-5753	TCGA-PRAD	Prostate gland	Male	<u>54</u>	<u>4</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>8</u>	<u>15</u>	<u>1</u> ½ (3)				
BCCCC IDMINIS	1.102			TCGA-EJ-7123	TCGA-PRAD	Prostate gland	Male	<u>65</u>	<u>7</u>	<u>10</u>	<u>16</u>	7	2	<u>8</u>	<u>15</u>	<u>1</u> <u>\$</u> (3)				
☐ HTSeq - Counts	58			TCGA-EJ-7312	TCGA-PRAD	Prostate gland	Male	<u>52</u>	<u>4</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>8</u>	<u>13</u>	<u>1</u> 호(1)				
HISeq-11-ton	551	551	551	551			F	TCGA-XK-AAJ3	TCGA-PRAD	Prostate gland	Male	<u>52</u>	<u>4</u>	<u>5</u>	<u>16</u>	<u>5</u>	1	<u>8</u>	<u>13</u>	<u>1</u> ½ (1)
☐ HTSeq - FPKM-UQ	551		F •	TCGA-CH-5763	TCGA-PRAD	Prostate gland	Male	<u>56</u>	4	<u>5</u>	<u>16</u>	<u>5</u>	2	<u>8</u>	<u>16</u>	<u>1</u> ½ (4)				
·			F	TCGA-HC-8259	TCGA-PRAD	Prostate gland	Male	<u>62</u>	<u>6</u>	<u>10</u>	<u>16</u>	<u>7</u>	1		<u>15</u>	<u>1</u> ½ (3)				
∨ Data Format			F •	TCGA-HC-7748	TCGA-PRAD	Prostate gland	Male	<u>54</u>	4	<u>5</u>	<u>16</u>	<u>5</u> -	1	8		<u>1</u> • (3)				
□ txt	2.755			TCGA-KC-A4BI	TCGA-PRAD	Prostate gland	Male	<u>58</u>	<u>5</u>	<u>5</u>	<u>16</u>	<u>7</u>	<u>2</u>		15 13	1 ½ (3)				

Project: TCGA-PRAD (Prostate Adenocarcinoma)



Outline

- Example: IncRNAs in prostate cancer in relation to Gleason score
 - Input Data preparation
 - MA Plots
 - Dimensionality Reduction
 - PCA
 - t-SNE
 - Statistical distributions
 - Poisson
 - Negative binomial
 - Statistical significance
 - *p*-values
 - Significant genes









Prostate cancer – read counts

499 patients (samples)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	:16_	17	18	19	20
	TCG	TCG	TCG	TCG	TCG/															
1 ENSG00000250529	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2 ENSG00000223985	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
3 ENSG00000257845	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4 ENSG00000216560	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5 ENSG00000257378	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	1	0
6 ENSG00000226733	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
7 ENSG00000215231	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8 ENSG00000231210	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0	0	2	0	0	0
9 ENSG00000281769	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
10 ENSG00000229005	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11 ENSG00000258548	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
12 ENSG00000249396	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0
13 ENSG00000276707	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14 ENSG00000278060	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0
15 ENSG00000228262	0	0	0	0	0	0	8	0	0	1	0	6	0	1	3	0	2	0	1	2
16 ENSG00000233639	0	0	0	0	0	0	3	0	0	0	0	2	0	0	2	0	0	0	0	0









Prostate cancer – clinical data

	1	2	3
	sample_ID	sample_file	gleason
1	'TCGA-2A-A8VL-01A-21R-A37L-07'	'TCGA-2A-A8VL-01A-21R-A37L-07'	'low'
2	'TCGA-2A-A8VO-01A-11R-A37L-07'	'TCGA-2A-A8VO-01A-11R-A37L-07'	'low'
3	'TCGA-2A-A8VT-01A-11R-A37L-07'	'TCGA-2A-A8VT-01A-11R-A37L-07'	'high'
4	'TCGA-2A-A8VV-01A-11R-A37L-07'	'TCGA-2A-A8VV-01A-11R-A37L-07'	'low'
5	'TCGA-2A-A8VX-01A-11R-A37L-07'	'TCGA-2A-A8VX-01A-11R-A37L-07'	'high'
6	'TCGA-2A-A8W1-01A-11R-A37L-07'	'TCGA-2A-A8W1-01A-11R-A37L-07'	'high'
7	'TCGA-2A-A8W3-01A-11R-A37L-07'	'TCGA-2A-A8W3-01A-11R-A37L-07'	'high'
8	'TCGA-2A-AAYF-01A-11R-A410-07'	'TCGA-2A-AAYF-01A-11R-A410-07'	'low'
9	'TCGA-2A-AAYO-01A-11R-A410-07'	'TCGA-2A-AAYO-01A-11R-A41O-07'	'low'
10	'TCGA-2A-AAYU-01A-11R-A410-07'	'TCGA-2A-AAYU-01A-11R-A410-07'	'low'
11	'TCGA-4L-AA1F-01A-11R-A410-07'	'TCGA-4L-AA1F-01A-11R-A410-07'	'high'
12	'TCGA-CH-5737-01A-11R-1580-07'	'TCGA-CH-5737-01A-11R-1580-07'	'high'
13	'TCGA-CH-5738-01A-11R-1580-07'	'TCGA-CH-5738-01A-11R-1580-07'	'low'
14	'TCGA-CH-5739-01A-11R-1580-07'	'TCGA-CH-5739-01A-11R-1580-07'	'low'
15	'TCGA-CH-5740-01A-11R-1580-07'	'TCGA-CH-5740-01A-11R-1580-07'	'low'
16	'TCGA-CH-5741-01A-11R-1580-07'	'TCGA-CH-5741-01A-11R-1580-07'	'high'
17	'TCGA-CH-5743-01A-21R-1580-07'	'TCGA-CH-5743-01A-21R-1580-07'	'low'
18	'TCGA-CH-5744-01A-11R-1580-07'	'TCGA-CH-5744-01A-11R-1580-07'	'high'
19	'TCGA-CH-5745-01A-11R-1580-07'	'TCGA-CH-5745-01A-11R-1580-07'	'low'
20	'TCGA-CH-5746-01A-11R-1580-07'	'TCGA-CH-5746-01A-11R-1580-07'	'low'









Dimensionality Reduction

- Prostate cancer count table
 - 500 features
 - 499 samples
- Lot of features to handle in transcriptomic data
- Example: Pasilla dataset
 - 14,599 features
 - 7 samples
- For visualization and analysis purposes it is good to reduce the dimensionality. Possible techniques:
 - PCA
 - t-SNE









PCA

- **Principal component analysis** (PCA) is a statistical procedure that uses an orthogonal transformation to convert a set of observations of possibly correlated variables (entities each of which takes on various numerical values) into a set of values of linearly uncorrelated variables called principal components.
- This transformation is defined in such a way that the first principal component has the largest possible variance (that is, accounts for as much of the variability in the data as possible), and each succeeding component in turn has the highest variance possible under the constraint that it is orthogonal to the preceding components.
- The resulting vectors (each being a linear combination of the variables and containing n observations) are an uncorrelated orthogonal basis set. PCA is sensitive to the relative scaling of the original variables.









PCA – MATLAB

```
coeff = pca(X)
```

- returns the principal component coefficients, also known as loadings, for the n-by-p data matrix X.
- Rows of X correspond to observations and columns correspond to variables. There are n observations and p variables.
- The coefficient matrix is p-by-p. Each column of <code>coeff</code> contains coefficients for one principal component, and the columns are in descending order of component variance. By default, <code>pca</code> centers the data and uses the singular value decomposition (SVD) algorithm.





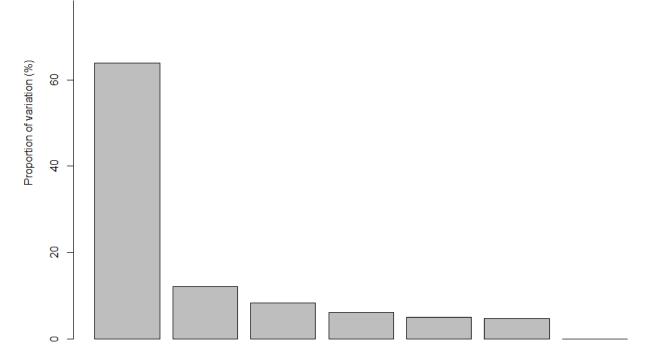




PCA – Scree plot

Importance of components:

```
PC1
                                   PC2
                                             PC3
                                                      PC4
                                                               PC5
                                                                        PC6
                                                                                   PC7
                       63.3517 27.6242 22.87721 19.44818 17.72155 17.10358 2.966e-13
Standard deviation
Proportion of Variance
                       0.6386
                                0.1214
                                        0.08328
                                                  0.06018
                                                           0.04997
                                                                    0.04655 0.000e+00
Cumulative Proportion
                        0.6386
                                0.7600
                                         0.84330
                                                  0.90348
                                                           0.95345
                                                                    1.00000 1.000e+00
```



Principal component (PC), 1-7



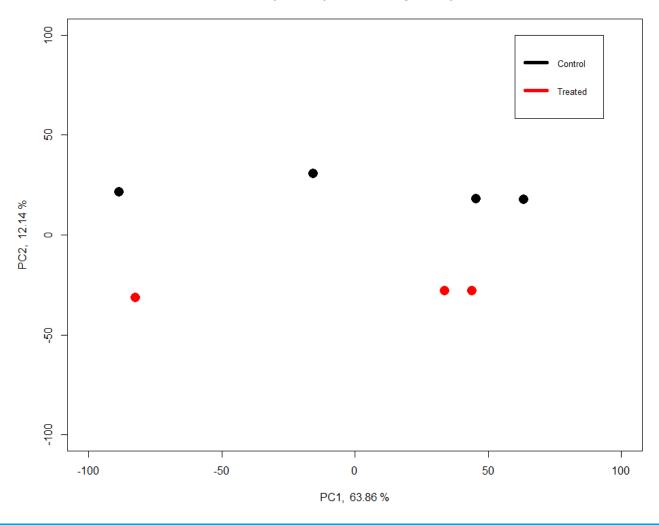






PCA – bi-plot PC1 and PC2

Principal components analysis bi-plot





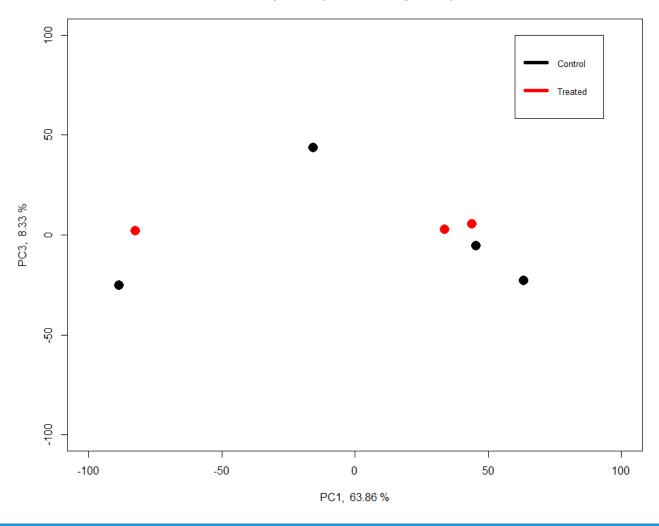






PCA – bi-plot PC1 and PC3

Principal components analysis bi-plot





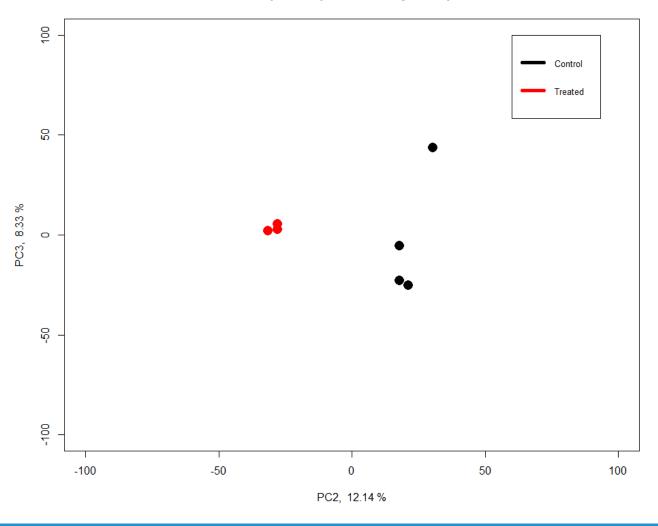






PCA – bi-plot PC2 and PC3

Principal components analysis bi-plot



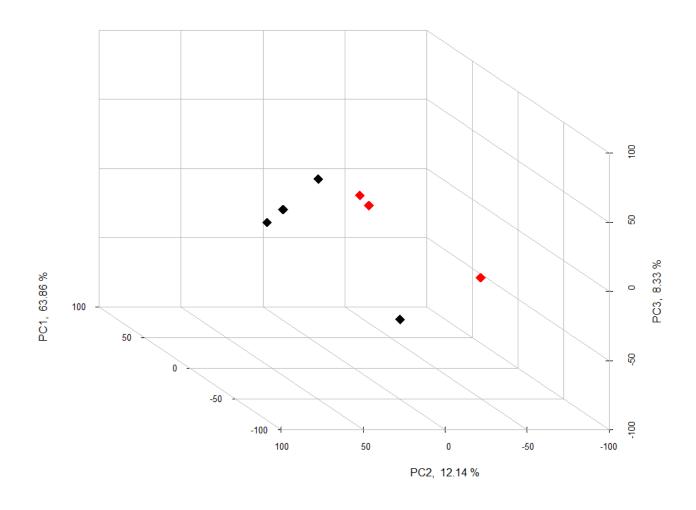








PCA – tri-plot PC1, PC2 and PC3











t-SNE

- It is a nonlinear dimensionality reduction technique well-suited for embedding high-dimensional data for visualization in a low-dimensional space of 2 or 3 dimensions.
- Specifically, it models each high-dimensional object by a 2D or 3D point in such a way that similar objects are modeled by nearby points and dissimilar objects are modeled by distant points with high probability.









t-SNE

- The t-SNE algorithm comprises two main stages:
 - 1. t-SNE constructs a probability distribution over pairs of high-dimensional objects in such a way that similar objects have a high probability of being picked while dissimilar points have an extremely small probability of being picked.
 - 2. t-SNE defines a similar probability distribution over the points in the low-dimensional map, and it minimizes the Kullback–Leibler divergence between the two distributions with respect to the locations of the points in the map.
- Hyper-parameter: perplexity
 - It is basically the effective number of neighbors for any point, and t-SNE works relatively well for any value between 5 and 50. Larger perplexities will take more global structure into account, whereas smaller perplexities will make the embeddings more locally focused.



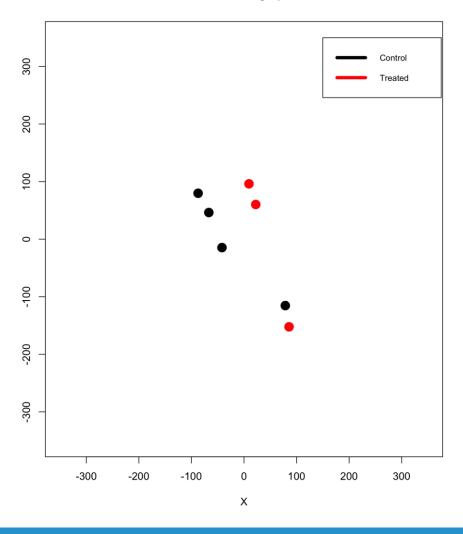






t-SNE bi-plot

t-SNE embedding bi-plot



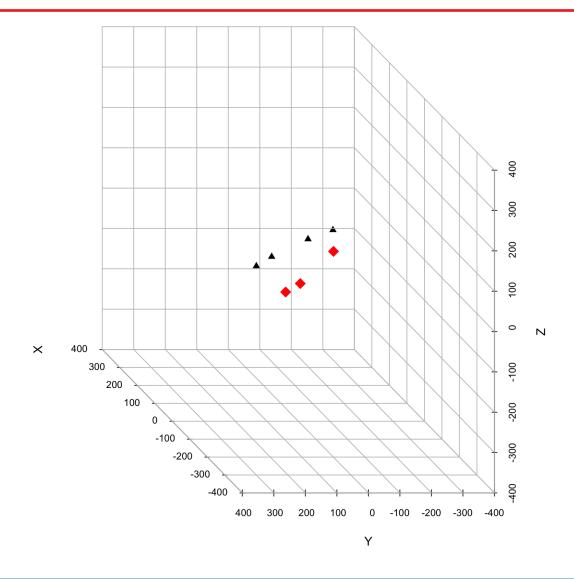








t-SNE tri-plot











t-SNE — MATLAB

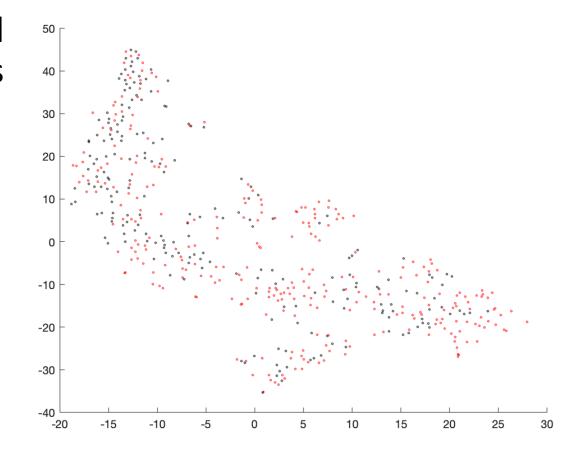
Y = tsne(X)

returns a matrix of two-dimensional embeddings of the high-dimensional rows of X.

X - Data points
specified as an n-by-m matrix, where each
row is one m-dimensional point.

Y - Embedded points

returned as an n-by-NumDimensions matrix. Each row represents one embedded point.











Fold Change

- Fold change is a measure describing how much a quantity changes between an original and a subsequent measurement.
- It is defined as the ratio between the two quantities; for quantities A and B, then the fold change of B with respect to A is B/A.
- In our case, we define Fold Change as:

```
foldChange = meanHigh ./ meanLow;
```

- You can look at the difference of the gene expression among two conditions, by calculating the fold change (FC) for each gene, i.e. the ratio between the counts in the high gleason group over the counts in the low gleason group.
- Generally these ratios are considered in the \log_2 scale, so that any change is symmetric with respect to zero.
- A ratio of 1/2 or 2/1 corresponds to -1 or +1 in the log scale.

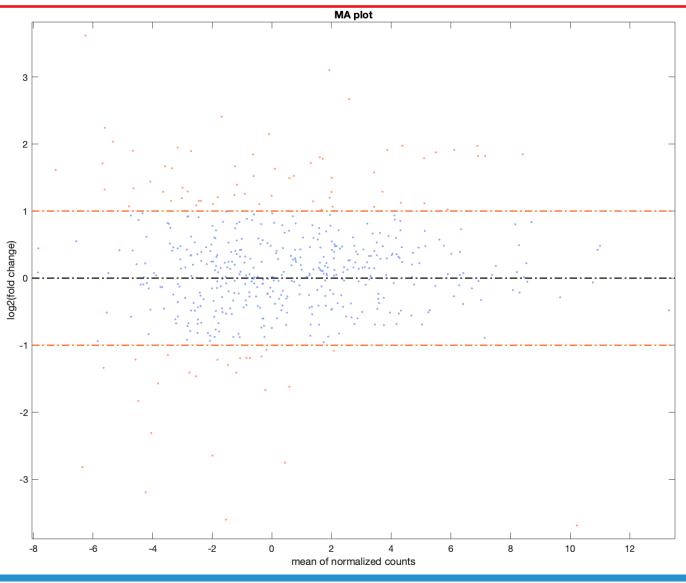








MA plot











Hypothesis Testing

- Based on a count table, we want to detect differentially expressed genes between different conditions.
 - How can we detect genes for which the counts of reads change between conditions more systematically than as expected by chance?
- We would like to use statistical testing to decide whether, for a given gene, an observed difference in read counts is significant, that is, whether it is greater than what would be expected just due to natural random variation.
- Null hypothesis H_0 :
 - ullet the gene g is not differentially expressed between the conditions
- Alternative hypothesis H_1 :
 - the gene g is differentially expressed between the conditions









Hypothesis Testing

- How to quantify the difference?
- The statistical tests do not give a simple answer of whether the hypothesis is true or not. What a statistical test determines is how likely that null hypothesis is to be true.
- After a test statistic is computed, it is often converted to a p-value. Then the difference is quantified in terms of the p-value.
- If the p-value is small then the null hypothesis is deemed to be untrue and it is rejected in favour of the alternative.
- The p-value is the probability of seeing a result as extreme or more extreme than the observed data, when the null hypothesis is true.
- It is a usual convention in biology to use a critical p-value of 0.05.









Type of errors in tests

Table of error	types	Null hypothesis (<i>H</i> ₀) is						
		True	False					
Decision about null hypothesis (<i>H</i> ₀)	Don't reject	Correct inference (true negative) (probability = 1 - α)	Type II error (false negative) (probability = β)					
	Reject	Type I error (false positive) (probability = α)	Correct inference (true positive) (probability = $1 - \beta$)					



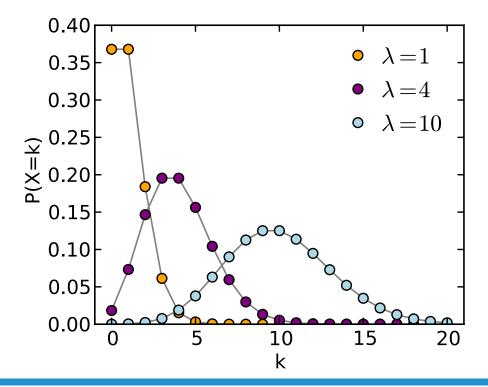






Poisson distribution

• The **Poisson distribution** is a discrete probability distribution that expresses the probability of a given number of events occurring in a fixed interval of time or space if these events occur with a known constant rate and independently of the time since the last event.



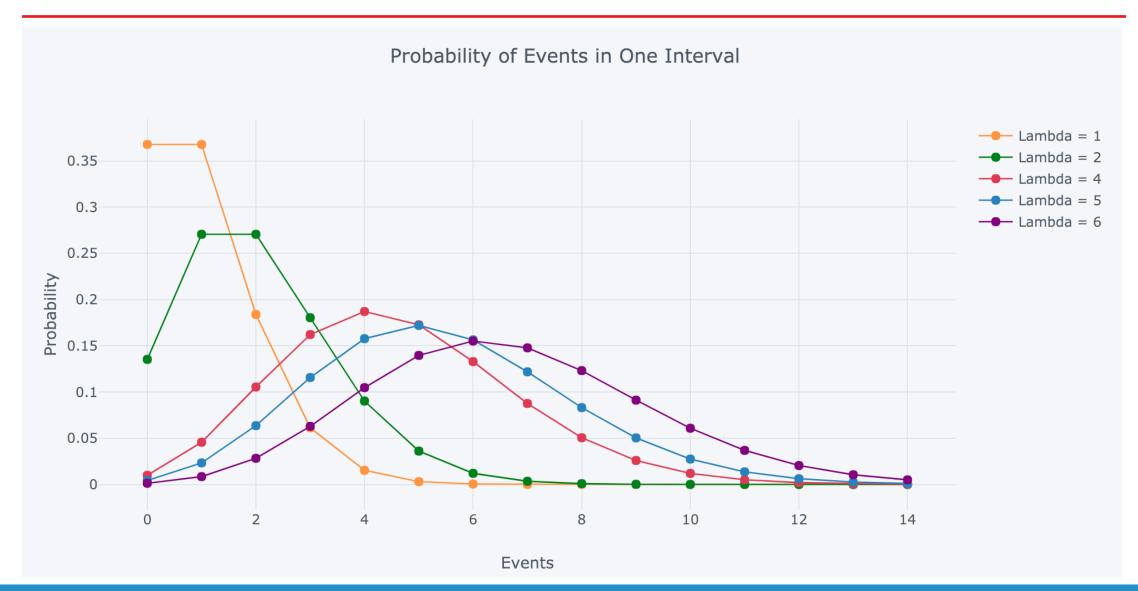








Poisson distribution











Poisson distribution

• We denote Poisson distribution with

$$Pois(\lambda)$$

- $\lambda \in \mathbb{R}^+$ is the *rate*
- Probability mass function (pmf)

$$f(k; \lambda) = \Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Mean

$$\mu = \lambda$$

Variance

$$\sigma^2 = \lambda$$









Negative binomial distribution

- The **negative binomial distribution** is a discrete probability distribution of the number of successes in a sequence of independent and identically distributed Bernoulli trials before a specified (non-random) number of failures (denoted r) occurs.
- For example, if we define a 1 as failure, all non-1s as successes, and we throw a dice repeatedly until 1 appears the 3^{rd} time (r = 3 failures), then the probability distribution of the number of non-1s that appeared will be a negative binomial distribution.









Negative binomial distribution

• We denote negative binomial distribution with:

- r > 0 is the number of failures until the experiment is stopped
- $p \in (0,1)$ is the probability of success for each experiment
- k is the number of successes
- Probability mass function (pmf)

$$f(k;r,p) = \Pr(X = k) = {k+r-1 \choose k} (1-p)^r p^k$$

Mean

$$\mu = \frac{pr}{1 - p}$$

Variance

$$\sigma^2 = \frac{pr}{(1-p)^2} = \mu + \frac{\mu^2}{r}$$

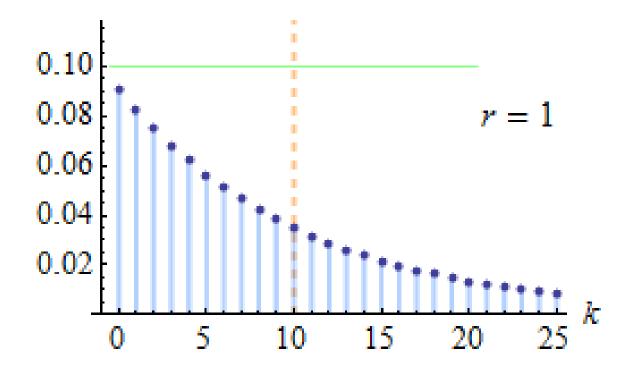








Negative binomial distribution











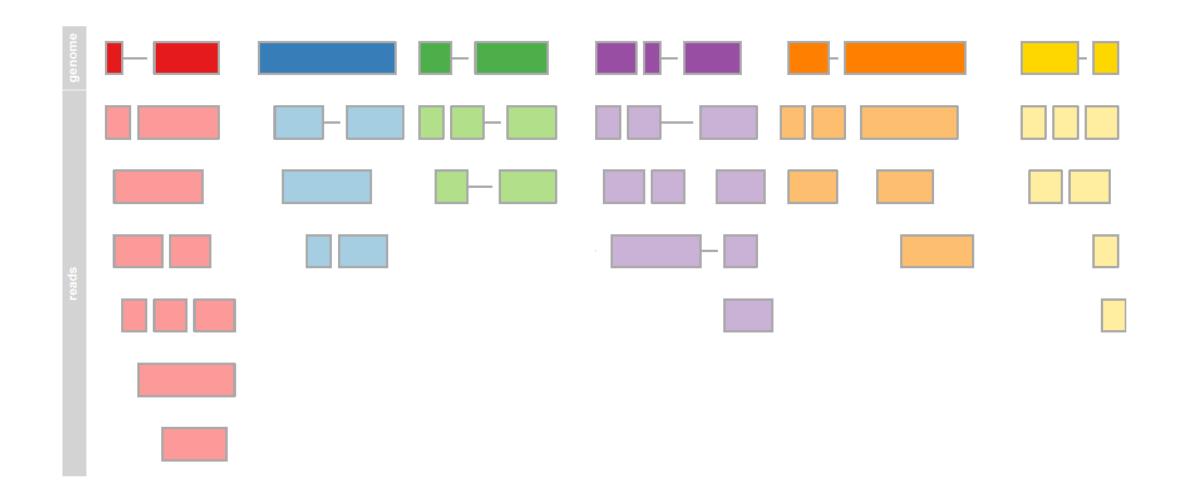




















 K_q : sequence reads assigned to a particular region g

 p_q : proportion of DNA fragments arising from the region g

 $K_q = 0,1,2,...,n$ (n + 1 possible discrete values)

n: total number of sequenced reads

We want to estimate $Pr(K_q = k)$



The probability of this is given by: $p^k(1-p)^{n-k}$

The number of ways of arranging k successes in n trials is:

$$\binom{n}{k} = \frac{n!}{k! (n-k)!}$$

Then, the probability of k successes in n independent trials is:

$$P(K_g = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

This is the Binomial distribution, which we denote $K_g \sim Bin(n, p)$









Poisson approximation

As the number of sequenced reads becomes large and the probability p shrinks, the Binomial distribution can be approximated by the Poisson distribution:

• split the pool of DNA fragments into a large number of n intervals, each with a small probability p of success and such that the total number of successes $K_q \sim \text{Bin}(n,p)$



ullet the number of successes, occurring in the interval $[0,\,n]$, occur independently and at a constant average rate $\lambda=np$. Then

$$\lambda \to np
n \to \infty
p \to 0$$

$$Bin(n,p) \to Pois(\lambda)
p \to 0$$

$$P(K_g = k) = \binom{n}{k} p^k (1-p)^{n-k}
= \binom{n}{k} \left(\frac{\lambda}{n}\right)^k \left(1-\frac{\lambda}{n}\right)^{n-k}
\approx \frac{\lambda^k}{k!} e^{-\lambda}$$

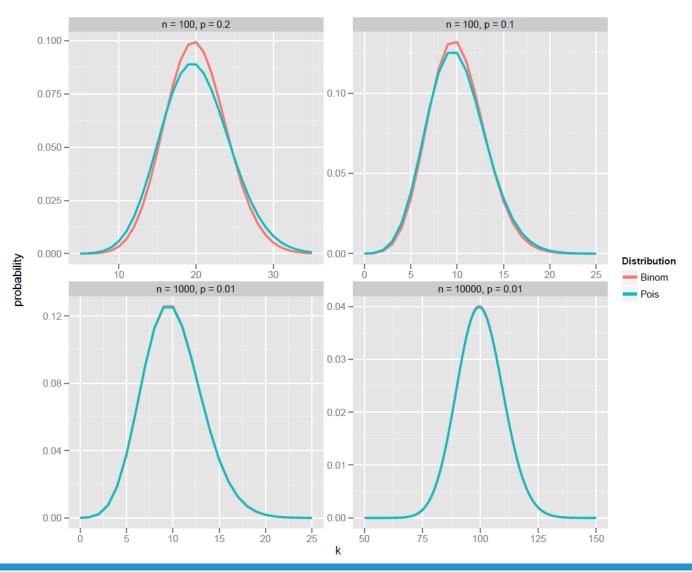
This is the **Poisson distribution** with parameters λ . We write $K_q \sim \mathsf{Pois}(\lambda)$.



















When the variance in a Poisson moldel is greater than the mean, the counts are said to be "overdispersed" with respect to a Poisson distribution.

To model overdispersed counts, the Poisson distribution model can be modified as

$$\mathcal{L}(K_a|\epsilon) \sim \mathsf{Pois}(\theta)$$
, with $\theta = \lambda \epsilon$

where ϵ is a nonnegative multiplicative random-effect term to model individual heterogeneity (Winkelmann 2008).

By placing a gamma distribution (see Box 2) prior on ϵ with $\alpha = \beta = r$, $\epsilon \sim \text{Gamma}(r,r)$, we have a negative binomial distribution for K_g , $K_g \sim \text{NB}(\alpha,p)$, parameterized by probability parameter $p = \lambda/(\lambda + r)$ and dispersion parameter $\alpha = r$ (see Box 3). The mean and variance is given by:

$$\mathsf{E}(K_g) = \lambda$$
 and $\mathsf{Var}(K_g) = \lambda + \phi \lambda^2$

where $\phi = 1/r$ is the inverse dispersion parameter. Thus $Var(K_g) \ge \mathsf{E}(K_g)$ and we obtain a model for overdispersed counts. Notice that, as ϕ decreases to 0, the variance of K_g approaches the usual Poisson variance λ (i.e. np_g).









Biological noise: overdispersion

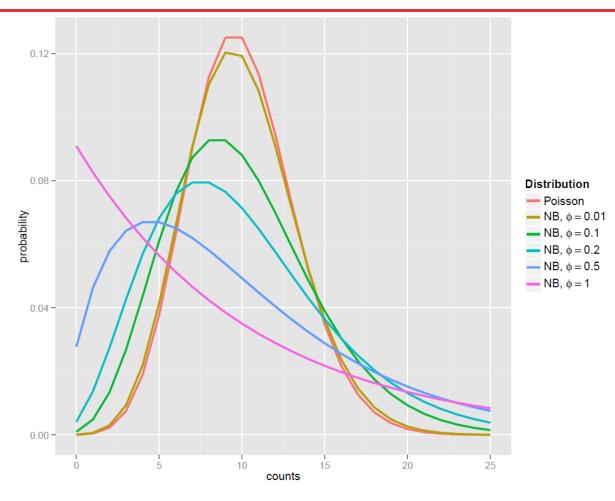


Figure 24: Poisson and negative binomial distributions, all with a mean value equal to 10. As the dispersion parameter, ϕ goes to zero, the negative binomial distribution converges to a Poisson. The discrete probabilities are jointed by lines for easing the visualization.









Bioinformatics Toolbox - nbintest()

- test = nbintest(X,Y) performs a hypothesis test that two independent samples of short-read count data, in each row of X and Y, come from distributions with equal means under the assumptions that:
 - Short-read counts are modeled using the negative binomial distribution.
 - Variance and mean of data in each row are linked through a regression function along all the rows.
- test is a NegativeBinomialTest object with two-sided p-values stored in the pValue property.
- Use this function when you want to perform an unpaired hypothesis test for short-read count data (from high-throughput assays such as RNA-Seq or ChIP-Seq) with small sample sizes (in the order of tens at most). For instance, use this function to decide if observed differences in read counts between two conditions are significant for given genes.









Bioinformatics Toolbox - nbintest()

'VarianceLink'

Linkage type between the variance and mean

- ➤ 'LocalRegression' The variance is the sum of the shot noise term (mean) and a locally regressed nonparametric smooth function of the mean as described in. This option is the default. Use this option if your data is overdispersed and has more than 1000 rows (genes).
- > 'Constant' The variance is the sum of the shot noise term (mean) and a constant multiplied by the squared mean. This method uses all the rows in the data to estimate the constant. Use this option if your data is overdispersed and has less than 1000 rows.
- ➤ 'Identity' The variance is equal to the mean as described in. Counts are therefore modeled by the **Poisson distribution** individually for each row of X and Y. Use this option if your data has few genes and the regression between the variance and mean is not possible because of very small number of samples or replicates. This option is not recommended for overdispersed data.

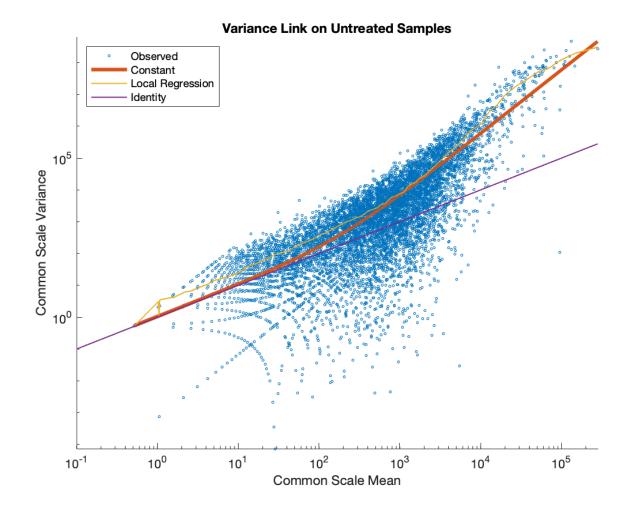


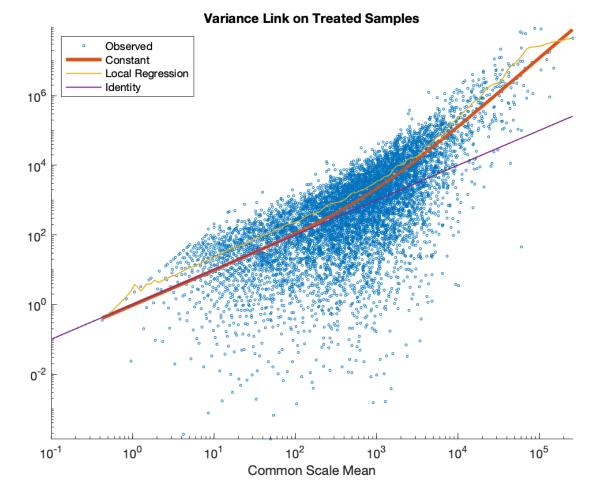






Variance Link – pasilla





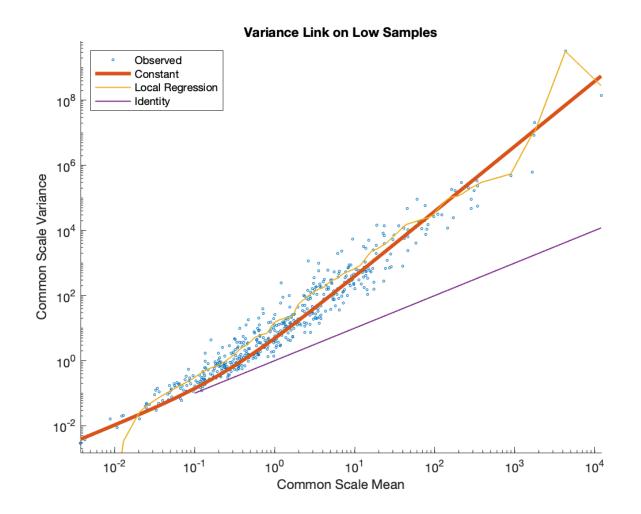


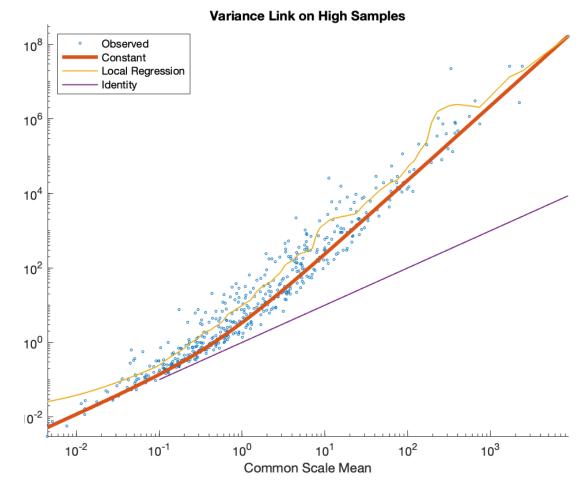






Variance Link – prostate cancer













p-values

- The output of nbintest includes a vector of p-values. A p-value indicates the probability that a change in expression as strong as the one observed (or even stronger) would occur under the null hypothesis, i.e. the conditions have no effect on gene expression.
- In the histogram of the p-values we observe an enrichment of low values (due to differentially expressed genes), whereas other values are uniformly spread (due to non-differentially expressed genes).
- The enrichment of values equal to 1 are due to genes with very low counts.

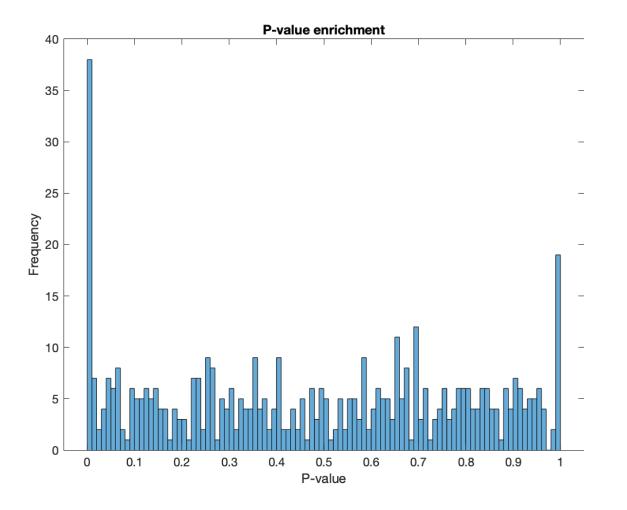


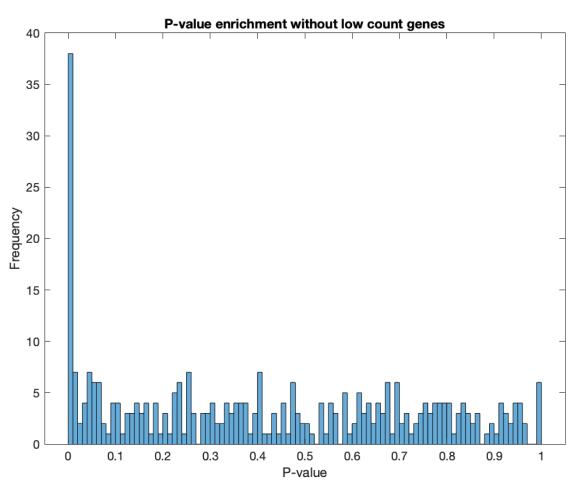






p-values histogram













Adjusted p-values

• Use mafdr function for obtaining adjusted p-values

```
FDR = mafdr(PValues)
```

returns FDR that contains a positive false discovery rate (pFDR) for each entry in PValues using the procedure introduced by Storey (2002).

PValues contains one p-value for each feature (for example, a gene) in a data set.

Optional parameter 'BHFDR' — Flag to use linear step-up procedure

Flag to use the linear step-up procedure introduced by Benjamini and Hochberg (1995), specified as the comma-separated pair consisting of 'BHFDR' and true or false.

The default value is false, that is, the function uses the procedure introduced by Storey (2002).









Benjamini-Hochberg adjustment

- The **Benjamini-Hochberg** (BH) adjustment is a statistical method that provides an adjusted p-value answering the following question: what would be the fraction of false positives if all the genes with adjusted p-values below a given threshold were considered significant?
- Set a threshold of 0.1 for the adjusted p-values, equivalent to consider a 10% false positives as acceptable, and identify the genes that are significantly expressed by considering all the genes with adjusted p-values below this threshold.
- We define a significant gene if adjusted p-value is less than 0.1

```
% compute the adjusted P-values (BH correction)
padj = mafdr(tLocal.pValue, 'BHFDR', true);
% add to the existing table
geneTable.pvalue = tLocal.pValue;
geneTable.padj = padj;
% create a table with significant genes
sig = geneTable.padj < 0.1;</pre>
```









Significant Genes

	1	2	3	4	5	6	7
	meanBase	meanHigh	meanLow	foldChange	log2FC	pvalue	padj
1 ENSG00000269900	2.3177e+03	333.8243	4.3016e+03	0.0776	-3.6877	0	0
2 ENSG00000281131	412.3767	645.3852	179.3682	3.5981	1.8472	1.2308e-20	3.0771e
3 ENSG00000235123	171.2898	266.9165	75.6632	3.5277	1.8187	1.1820e-13	1.9700e
4 ENSG00000258162	147.3062	234.7539	59.8586	3.9218	1.9715	4.9304e-12	6.1630e
5 ENSG00000275173	0.6489	0.0988	1.1990	0.0824	-3.6015	4.4600e-11	4.4600e
6 ENSG00000226779	8.7947	15.1989	2.3904	6.3584	2.6687	1.5877e-09	1.3231e
7 ENSG00000235984	85.0298	134.3791	35.6805	3.7662	1.9131	1.0660e-08	7.6144e
8 ENSG00000225258	145.1493	226.2200	64.0785	3.5304	1.8198	1.2720e-07	7.9499e
9 ENSG00000253369	54.9525	86.3528	23.5521	3.6665	1.8744	2.7079e-07	1.5044e
10 ENSG00000183535	2.0214	0.5220	3.5209	0.1483	-2.7539	5.3606e-06	2.6803e
11 ENSG00000254695	1.2058	1.9680	0.4436	4.4364	2.1494	5.9909e-06	2.7232e
12 ENSG00000259457	434.1445	556.3867	311.9022	1.7838	0.8350	8.5056e-06	3.5440e
13 ENSG00000261538	25.7713	41.0953	10.4473	3.9336	1.9758	1.4281e-05	5.4927e
14 ENSG00000243081	6.2161	11.1360	1.2962	8.5914	3.1029	1.8614e-05	6.6478e
15 ENSG00000278626	297.2310	377.9431	216.5189	1.7455	0.8037	3.9420e-05	0.0013
16 ENSG00000254080	41.1710	63.8399	18.5020	3.4504	1.7868	8.6679e-05	0.0027









Significant Genes

	1	2	3	4	5	6	7
	meanBase	meanHigh	meanLow	foldChange	log2FC	pvalue	padj
10 ENSG00000183535	2.0214	0.5220	3.5209	0.1483	-2.7539	5.3606e-06	2.6803e
11 ENSG00000254695	1.2058	1.9680	0.4436	4.4364	2.1494	5.9909e-06	2.7232e
12 ENSG00000259457	434.1445	556.3867	311.9022	1.7838	0.8350	8.5056e-06	3.5440e
13 ENSG00000261538	25.7713	41.0953	10.4473	3.9336	1.9758	1.4281e-05	5.4927e
14 ENSG00000243081	6.2161	11.1360	1.2962	8.5914	3.1029	1.8614e-05	6.6478e
15 ENSG00000278626	297.2310	377.9431	216.5189	1.7455	0.8037	3.9420e-05	0.0013
16 ENSG00000254080	41.1710	63.8399	18.5020	3.4504	1.7868	8.6679e-05	0.0027
17 ENSG00000258077	0.4264	0.7175	0.1352	5.3060	2.4076	1.6657e-04	0.0049
18 ENSG00000233508	17.9213	28.3133	7.5293	3.7604	1.9109	2.4633e-04	0.0068
19 ENSG00000251152	2.9183	4.4752	1.3613	3.2875	1.7170	7.0543e-04	0.0186
20 ENSG00000280703	3.8910	6.0291	1.7529	3.4394	1.7822	9.5195e-04	0.0238
21 ENSG00000259443	3.6446	5.6643	1.6249	3.4860	1.8016	0.0012	0.0285
22 ENSG00000243479	63.0556	84.5346	41.5766	2.0332	1.0238	0.0020	0.0452
23 ENSG00000229807	11.5330	15.6013	7.4646	2.0900	1.0635	0.0023	0.0489
24 ENSG00000235736	0.1386	0.2201	0.0571	3.8537	1.9463	0.0023	0.0489
25 ENSG00000266120	0.1871	0.2949	0.0794	3.7155	1.8935	0.0033	0.0658
26 ENSG00000228055	0.0812	0.0272	0.1352	0.2015	-2.3114	0.0039	0.0714
27 ENSG00000251003	1.2448	1.8820	0.6076	3.0973	1.6310	0.0039	0.0714
28 ENSG00000260760	0.3651	0.1005	0.6297	0.1596	-2.6474	0.0045	0.0768
29 ENSG00000253643	12.4192	18.6023	6.2361	2.9830	1.5768	0.0043	0.0768
30 ENSG00000280623	1.0315e+04	8.6166e+03	1.2012e+04	0.7173	-0.4793	0.0054	0.0897









References

- MATLAB Documentation
 - RNA-Seq Example. URL: https://it.mathworks.com/help/bioinfo/ug/identifying-differentially-expressed-genes-from-rna-seq-data.html
- Wikipedia
 - Negative binomial distribution. URL: https://en.wikipedia.org/wiki/Negative-binomial-distribution
 - Poisson distribution. URL: https://en.wikipedia.org/wiki/Poisson distribution
- StatQuest with Josh Starmer
 - URL: https://www.youtube.com/channel/UCtYLUTtgS3k1Fg4y5tAhLbw
- Ignacio Gonzalez (2014), Tutorial Statistical analysis of RNA-Seq data. URL: http://www.nathalievialaneix.eu/doc/pdf/tutorial-rnaseg.pdf
- Anders, S., & Huber, W. (2010). Differential expression analysis for sequence count data. *Genome biology*, 11(10), R106.
- Graveley, B. R., Brooks, A. N., Carlson, J. W., Duff, M. O., Landolin, J. M., Yang, L., ... & Brown, J. B. (2011). The
 developmental transcriptome of Drosophila melanogaster. *Nature*, 471(7339), 473.
- Storey, J. D. (2002). A direct approach to false discovery rates. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 64(3), 479-498.
- Benjamini et al. Controlling the false discovery rate: a practical and powerful approach to multiple testing. 1995. Journal of the Royal Statistical Society, Series B57 (1):289-300.







